# ESDI: Entanglement Scheduling and Distribution in the Quantum Internet

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# Outline

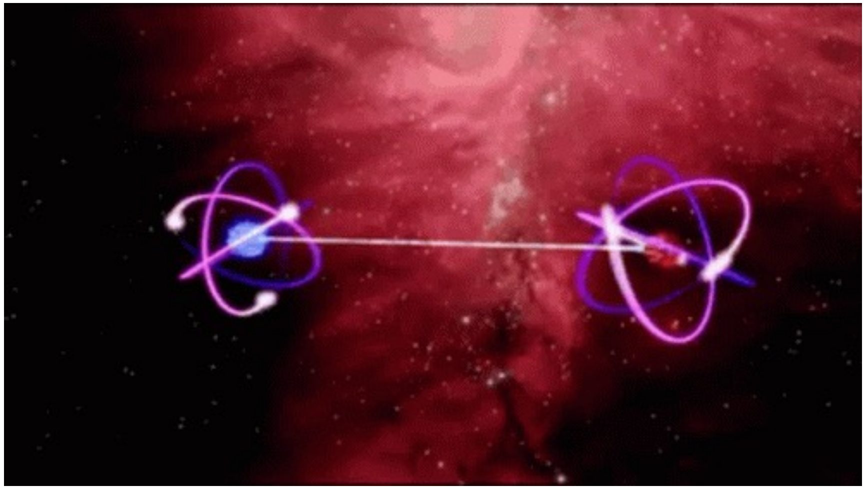
# A quantum network

A quantum network enables efficient and secure quantum communication based on **quantum entanglement.**

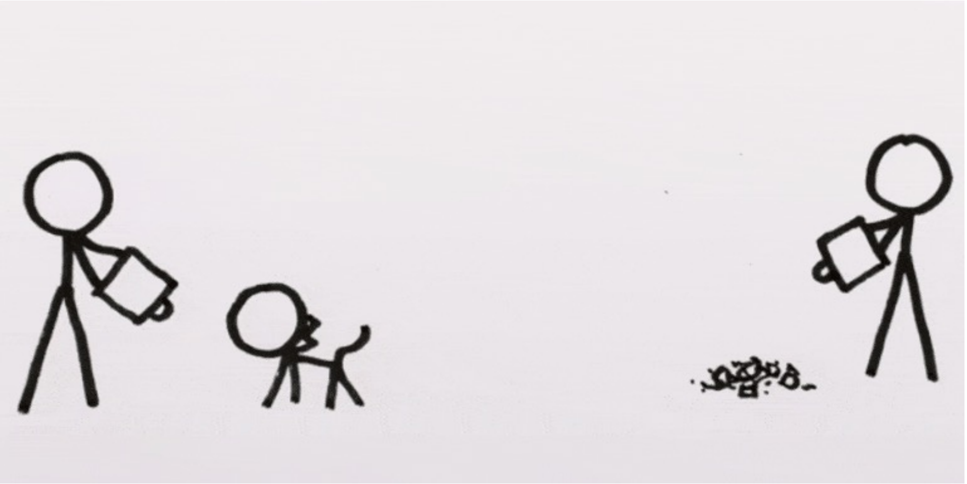
**Qubits**: quantum information = *quantum bits*

**Quantum entanglement of two qubits**

- Reveal both by revealing either
- Even separated by a large distance



(1) Quantum entanglement[1]



(2) Quantum teleportation[2]



[1] https://tenor.com/view/entanglement-quantum-entanglement-science-atoms-gif-17770432
[2] https://www.popularmechanics.com/science/a25699/how-quantum-teleportation-works/

# Entanglement generation and swapping


(a) Entanglement Generation

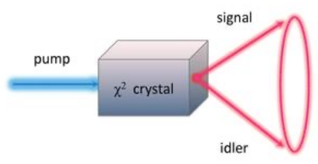
(b) Entanglement Swapping

(c) Entanglement Established

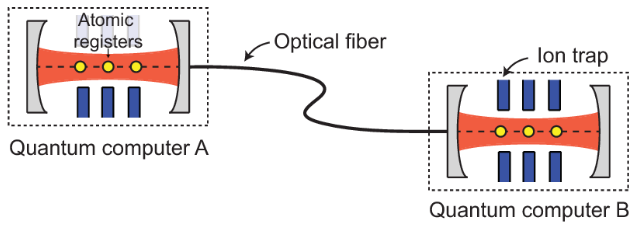
**In this way, each end-to-end entanglement is thus generated along an entanglement path in a quantum network.**

# Unique properties

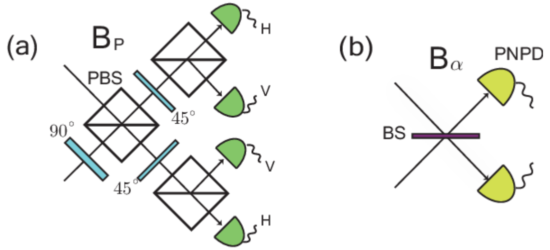
- **Probabilistic quantum operation in quantum networks:**




(1) Entanglement generation[1]

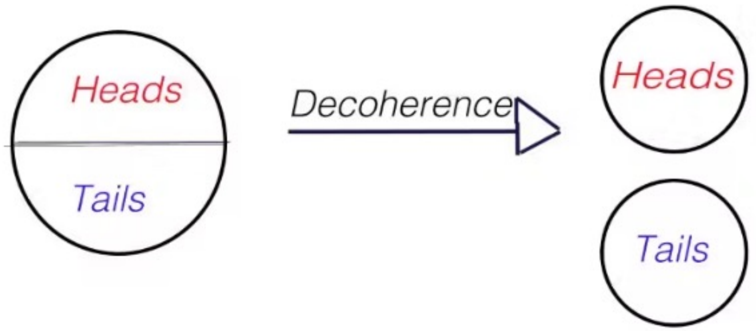



(2) Entanglement transmission[2]




(3) Entanglement swapping[3]

- **Decoherence in quantum applications:**




(4) Quantum decoherence[4]

[1] https://en.wikipedia.org/wiki/Spontaneous_parametric_down-conversion
[2] Kim, Tony Hyun. *An optical-fiber interface to a trapped-ion quantum computer*. Diss. Massachusetts Institute of Technology, 2011.
[3] Lee, Seung-Woo, and Hyunseok Jeong. "Bell-state measurement and quantum teleportation using linear optics: two-photon pairs, entangled coherent states, and hybrid entanglement." *arXiv preprint arXiv:1304.1214* (2013).
[4] https://hackernoon.com/decoherence-quantum-computers-greatest-obstacle-67c74ae962b6

# Quantum applications

- **Time-insensitive applications:**
  - Support long-term stream of entanglements
  - Require secure communications
  - E.g., Quantum key distribution (QKD)[1]

- **Time-sensitive applications:**
  - Complete tasks as quickly as possible
  - Avoid information decoherence
  - E.g., Distributed quantum computing (DQC)[2]

[1] https://www.drishtiias.com/daily-updates/daily-news-analysis/quantum-key-distribution-technology/print_manually
[2] https://www.nature.com/articles/s41566-020-00718-2

# Related work and limitations

- **Specialized quantum network topologies**
  - Repeater chain, lattices, star, and ring-like topologies
- **Entanglement routing[1,2,3]**
  - Bufferless assumption: entanglement decoherent after one time slot
- **Optimal remote entanglement distribution (ORED)[4]**
  - Buffered assumption: entangled qubits stored in quantum memories
  - Optimal EDR (entanglement distribution rate)
  - Only one source-destination (SD) pair
  - No scheduling consideration

[1] S. Shi and C. Qian, "Concurrent entanglement routing for quantum networks: Model and designs," in ACM SIGCOMM, 2020, pp. 62–75.
[2] Y. Zhao and C. Qiao, "Redundant entanglement provisioning and selection for throughput maximization in quantum networks," in IEEE INFOCOM, 2021, pp. 1–10.
[3] Y. Zeng, J. Zhang, J. Liu, Z. Liu, and Y. Yang, "Multi-entanglement routing design over quantum networks," in IEEE INFOCOM, 2022.
[4] W. Dai, T. Peng, and M. Z. Win, "Optimal protocols for remote entanglement distribution," in IEEE ICNC, 2020, pp. 1014–1019.

# Contributions

- **Problems**
  - A buffered quantum network
  - Multiple requests *(commodities)* for multiple SD pairs
  - Define the entanglement scheduling and distribution problem
- **A general framework for scheduling and distribution (ESDI)**
  - **ESDI-O**: commodities having demands but no deadlines
  - **ESDI-E**: commodities having demands and deadlines
- **Data plane protocol design**
- **Evaluations**

# Outline

Background and Motivation

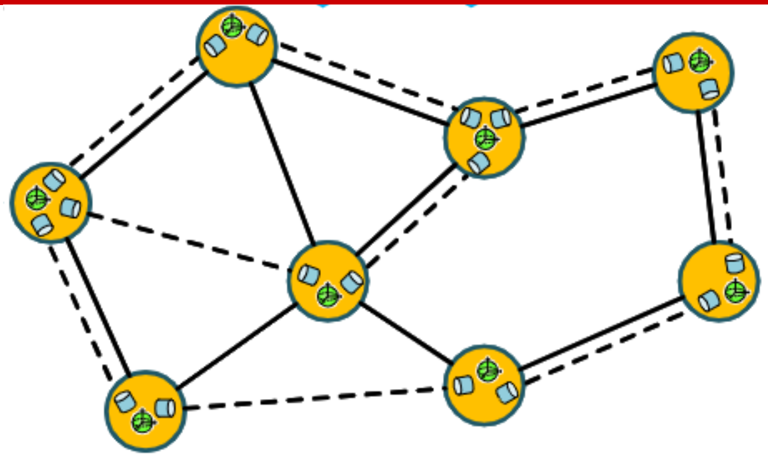
Quantum Network Model

Solution Design

Performance Evaluation

Discussions, Future Work and Conclusions

# Quantum Network Model



An undirected graph $G = (V, E)$

- $V$: the set of quantum repeaters
- $E$: the set of physical channels (links) between repeaters
- $c_e$: the number of ebits that can be generated along each link in unit time
- $p_e$: the probability of successfully generating an elementary ebit
- $q_\mathrm{v}$: the probability of successfully performing swapping

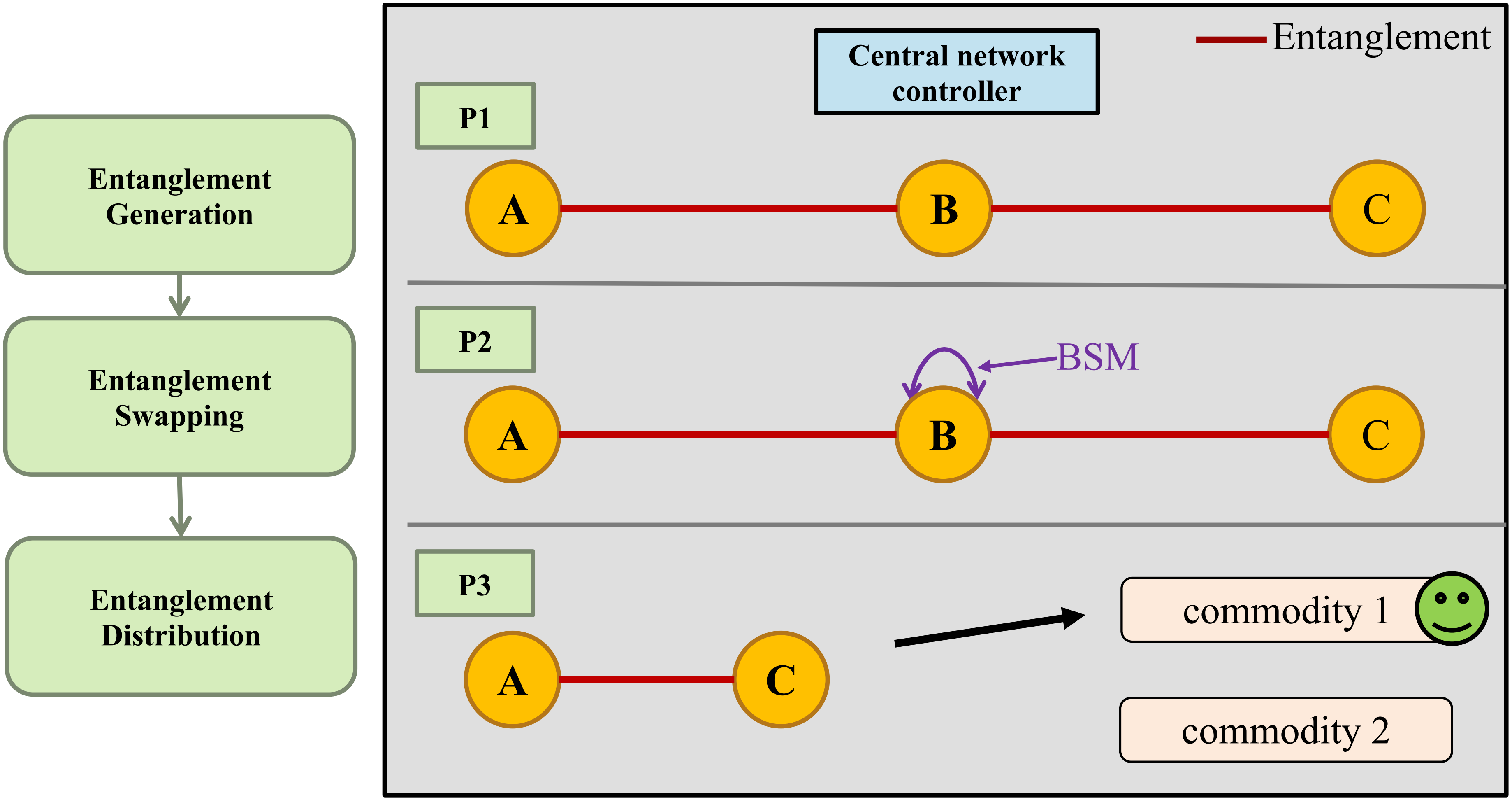
*Entangled qubit pairs as* ***ebits***

*Ebits generated along a physical channel as* ***elementary ebits***

# Quantum Network Model

**A time-slotted quantum network with discrete time $T = \{1, 2, 3, \ldots\}$**

- Three phases in each time slot



# Goals

**Real-time Decisions** → **Goals** → **Commodities**

**Real-time Decisions**

- How to generate ebits over physical links
- How to swap ebits at intermediate repeaters
- How to distribute end-to-end ebits to commodities

**Commodities**

- Meet as many deadlines of commodities as possible
- Complete commodities without deadlines as fast as possible

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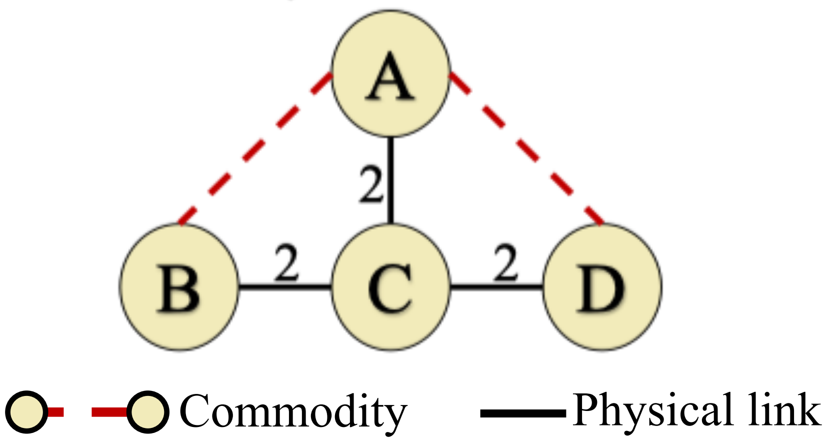
# A motivating example

*The demand of A-B = demand of A-D = 6 ebits. The EDR is 1 ebit per time slot.* $c_e = 2$.

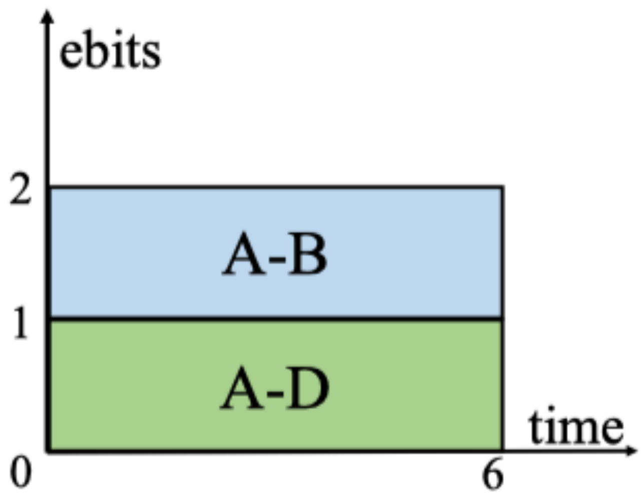



If commodities without deadlines:

**Average completion time will be reduced.**

If the deadline for commodity A-B is 4 and A-D is 6:

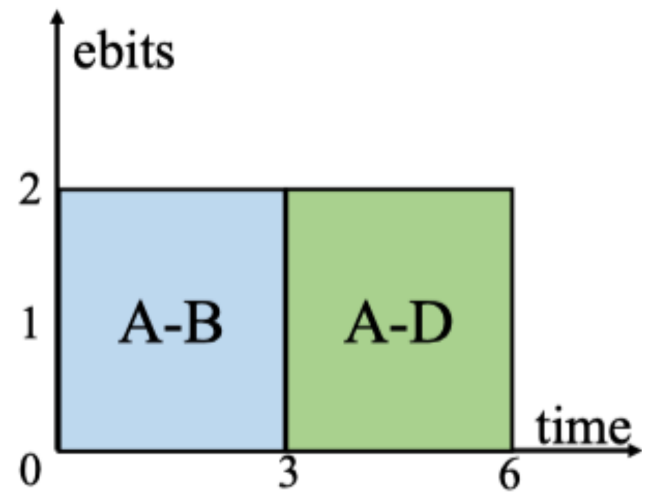
**100% improvement in deadline satisfaction ratio.**




(b) Fair Sharing

**Average completion time = 6**

**A-B Fail**




(c) Scheduling

**Average completion time = $\frac{6+3}{2}$ = 4.5**

**All Success**

# Problem Statement

**Definition 1.** Given a quantum network $G$ and commodities $Z = \bigcup_i Z_i$, a solution to the ***entanglement scheduling and distribution (ESDI)*** problem consists of three algorithms, $(\mathcal{A}_{gen}, \mathcal{A}_{swap}, \mathcal{A}_{dis})$ to perform the following tasks respectively:

- $\mathcal{A}_{gen}\ (S_T^0)$: In Phase-1 at time $T$, decide the number of ebits to attempt along physical link $e \in E$;
- $\mathcal{A}_{\text{swap}}(S_T^1)$: In Phase-2 at time $T$, given the number of ebits between node pairs $m{:}k$ and $k{:}n$ respectively, decide how many ebit pairs are used to swap for node pair $m{:}n$, for $\forall\ m, k, n \in V$;
- $\mathcal{A}_{dis}(S_T^2)$: In Phase-3 at time $T$, given the number of ebits between each SD pair $s{:}t \in U$, decide how many ebits are distributed to each commodity $z_j^i \in Z_i$.

# Multi-commodity remote entanglement distribution

## Question

Can we design a multi-commodity formulation (MRED)?

## Challenge

- Multiple SD pairs for scheduling in the quantum network.
- Each SD pair has multiple commodities arriving at different time slots.

## Idea

Can we apply classical task scheduling disciplines to quantum networks?

# Multi-commodity remote entanglement distribution

A linear programming problem extended from ORED

$$\text{(MRED)} \quad \text{find } \mathcal{F} \qquad (1)$$

$$\text{s.t.} \quad f_{m:n}^{m:k} = f_{m:n}^{k:n}, \quad \forall k, m, n \in V; \qquad (1a)$$

$$I(m{:}n) = \Omega(m{:}n), \ \forall m, n \in V, m{:}n \notin U; \qquad (1b)$$

$$I(s{:}t) \geq \Omega(s{:}t), \ \forall s{:}t \in U. \qquad (1c)$$

Two auxiliary functions $I(m{:}n)$ and $\Omega(m{:}n)$ are defined as:

$$I(m{:}n) \triangleq 1_{m:n} p_{mn} c_{mn} g_{m:n} + \sum_{k \in N \setminus \{m,n\}} \frac{q_k}{2} \left( f_{m:n}^{m:k} + f_{m:n}^{k:n} \right); \qquad (1d)$$

$$\Omega(m{:}n) \triangleq \sum\nolimits_{k \in N \setminus \{m,n\}} \left( f_{m:k}^{m:n} + f_{k:n}^{m:n} \right), \qquad (1e)$$

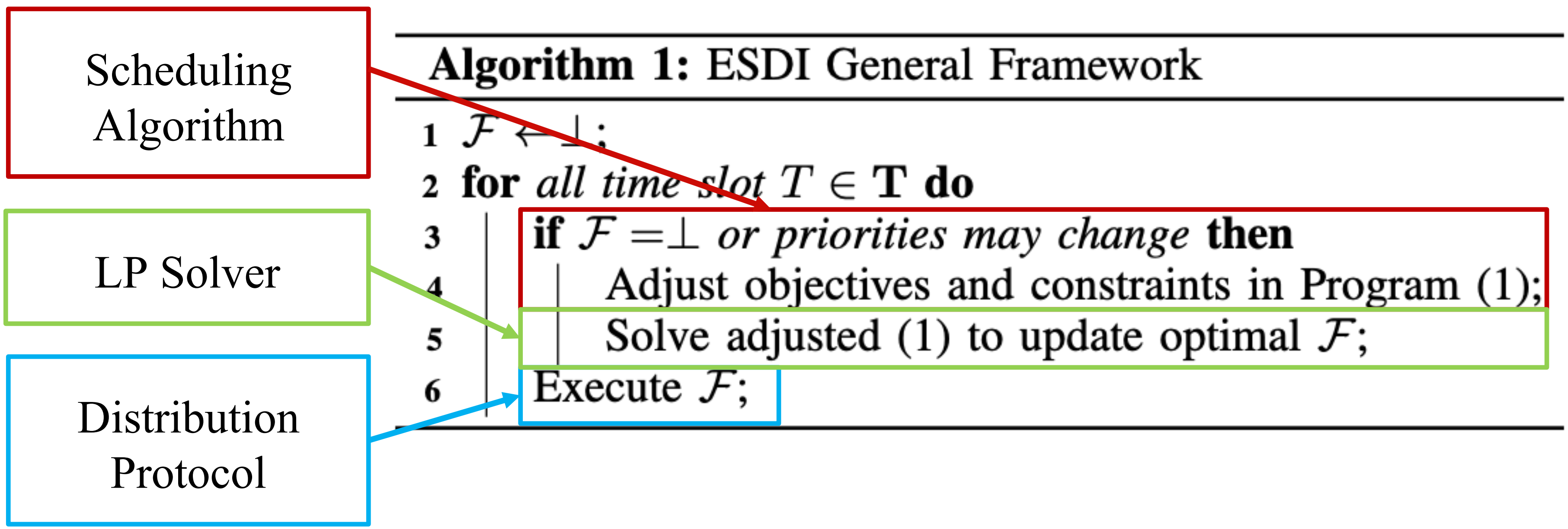
$\boldsymbol{\mathcal{F} = \{ f_{m:n}^{m:k} \geq 0 | m, k, n \in V \} \cup \{ g_{m:n} \in [0,1] | (m{:}n) \in E \}}$**:** the solution satisfying all constraints
$\boldsymbol{f_{m:n}^{m:k}}$: the number of ebits between $m{:}k$ being contributed to swapping with ebits between $k{:}n$.
$\boldsymbol{g_{m:n}}$: the number of ebits that would be attempted to be generated along physical link $m{:}n \in E$

**Theorem 1**. The optimal total EDR $\eta^*$ is upper bounded by $max_F\{\sum_{s:t}(I(s{:}t) - \Omega(s{:}t)) | F \text{ is feasible to } (1)\}$, and there exits a stationary ESDI protocol with expected total EDR equal to $\eta^*$.

# A General Framework for ESDI

**ESDI General Framework:**

1. *Scheduling* (prioritizing certain SD pairs)
2. *Work conservation* (maximizing network EDR)



# Scheduling Design for ESDI

- **Challenge 1**: commodities without deadlines
- **Goal**: minimize the average completion time of all commodities
- **Technique: Shortest job first (SJF)**

$$\text{(MRED-DC)} \quad \max \sum_{s:t \in U} \eta_{st} \tag{3}$$

$$\text{s.t.} \quad \eta_{s_i t_i} \Delta_j^i \geq \sum_{z_j^i \in P_c^i[l]} \Theta_j^i, \quad \forall s_i{:}t_i \in U_c, l = 1, 2, \ldots, |P_c^i|; \tag{3a}$$

$$\text{Constraints (1a)–(1e) and (2a).}$$

- **Challenge 2:** commodities with deadlines
- **Goal**: finish transmitting the information before irreversible errors happen
- **Technique: Earliest deadline first (EDF)**

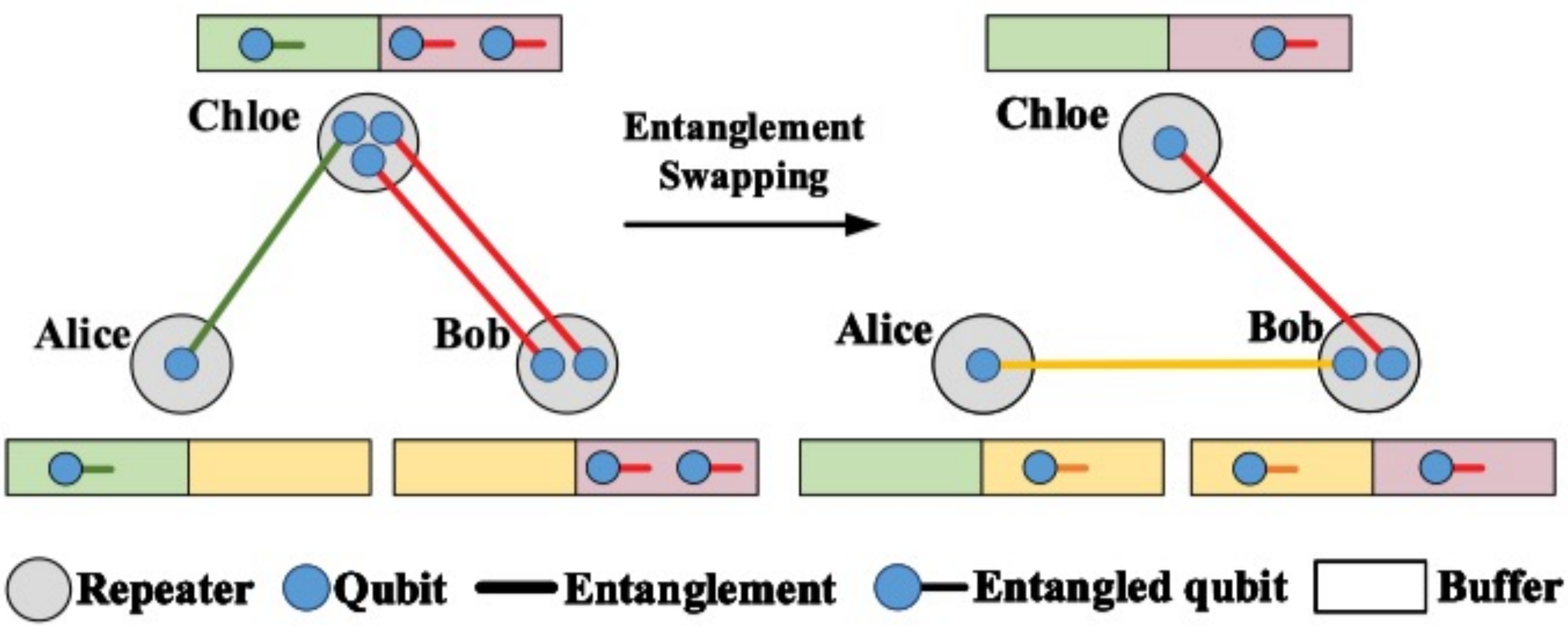
$$\text{(MRED-SP)} \quad \max \eta_1, \ \max \eta_2, \ \ldots, \ \max \eta_\kappa,$$

$$\max \sum_{s:t \in U} \eta_{st} \tag{2}$$

$$\text{s.t.} \quad \eta_{st} = I(s{:}t) - \Omega(s{:}t), \quad \forall s{:}t \in U; \tag{2a}$$

$$\text{Constraints (1a)–(1e).}$$

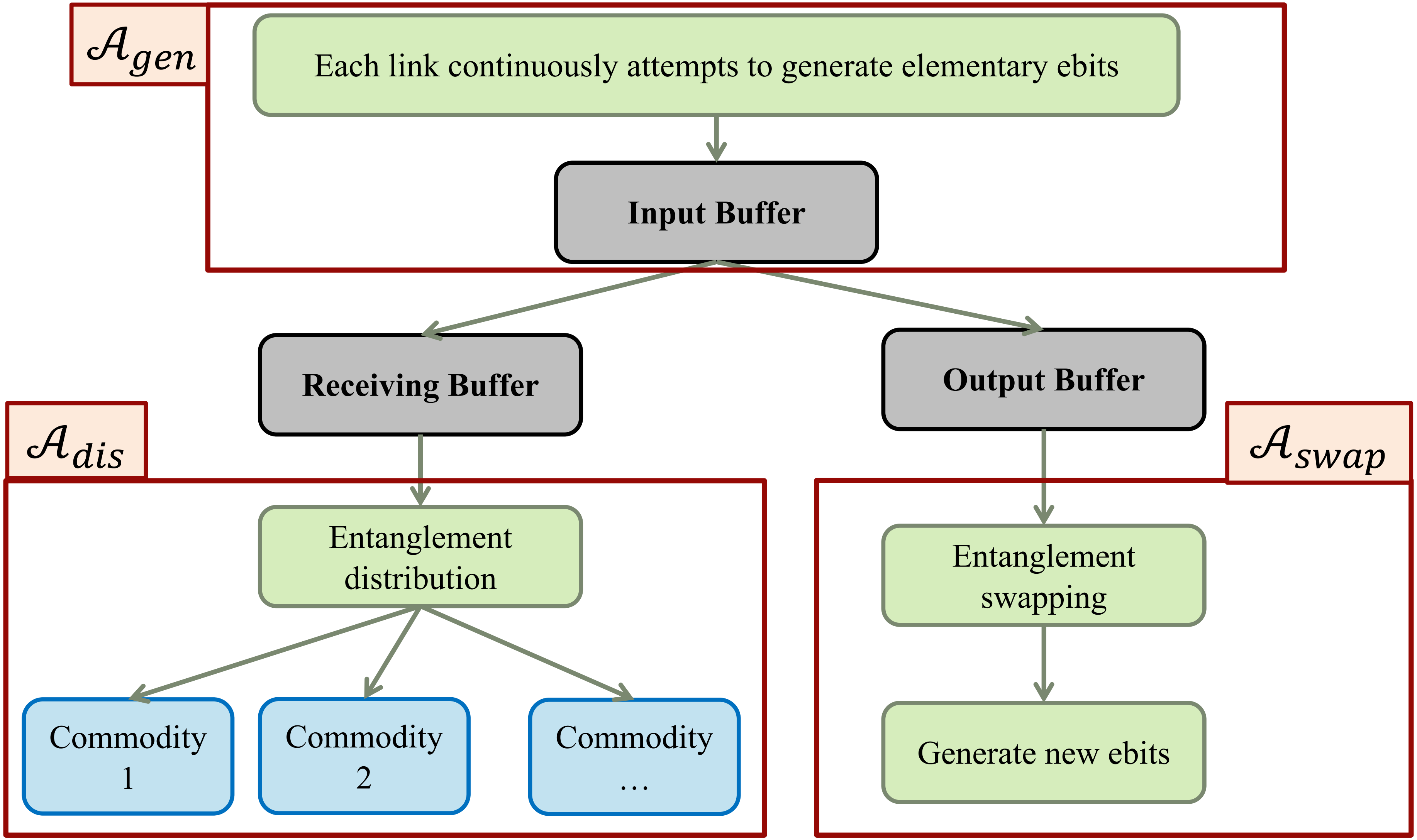
# Buffered quantum network




**Assumption:**

1. Each quantum repeater is equipped with sufficient quantum memories as buffers.

# Distribution Design for ESDI



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**Performance Evaluation**

Discussions, Future Work and Conclusions

# Evaluation Methodology

Generate graphs with 20 nodes and picks 1000 random SD pairs. $q_v = p_e = 0.9,\ c_e = [3, 10]$.
The following control plane algorithms were compared:

- **ESDI-B**: basic ESDI without scheduling as in **MRED**;
- **ESDI-O**: ESDI without deadlines in Algorithm 2;
- **ESDI-E**: ESDI with deadlines in Algorithm 3;
- **E2E-F**: fidelity-aware protocol [2] maximizing end-to-end EDR. We set fidelity as 1 since it is not considered;
- **QPASS**: QPASS protocol [3] trying to maximize end-to-end EDR for multiple SD pairs.

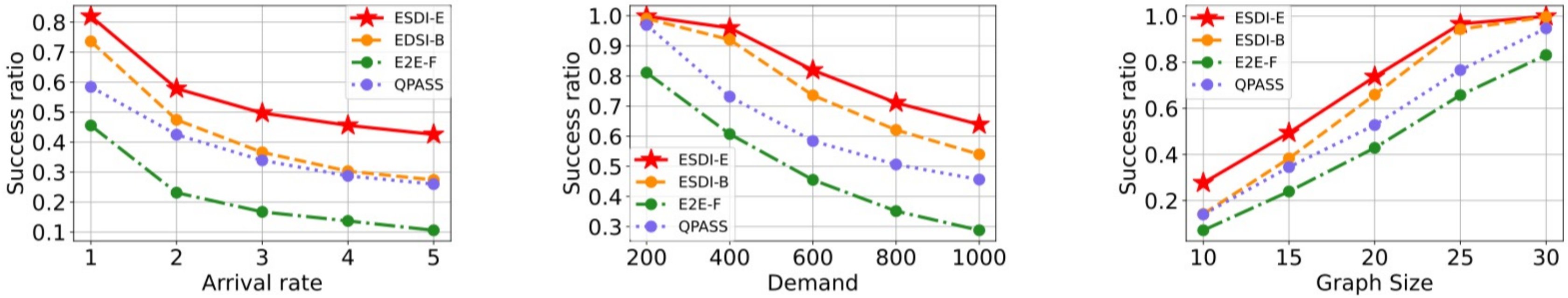
For QPASS and E2E-F, the number of paths K = 15 for Yen's algorithm.
For each commodity in default:
**Arrival rate** followed a Poisson Distribution with $\lambda = 1$ by default.
**Demands** followed an exponential distribution with mean of 600 ebits and a minimum demand of 100 ebits per commodity.
**Deadline** followed $\delta_j^i = a_j^i + \bar{\delta}_j^i \cdot d_j^i$, where $\bar{\delta}_j^i$ is a unit deadline following a uniform distribution in the range $[\mu - 0.1, \mu + 0.1]$.
**Scheduling length** $\kappa = 1$

**Metrics**:
**Success ratio**: the ratio of the number of commodities finished before their deadlines.
**Average completion time**: the average time between each commodity's arrival and completion when there is no deadline.



[1] W. Dai, T. Peng, and M. Z. Win, "Optimal protocols for remote entanglement distribution," in IEEE ICNC, 2020, pp. 1014–1019
[2] Y. Zhao, G. Zhao, and C. Qiao, "E2E fidelity aware routing and purification for throughput maximization in quantum networks," in IEEE INFOCOM, 2022.
[3] S. Shi and C. Qian, "Concurrent entanglement routing for quantum networks: Model and designs," in ACM SIGCOMM, 2020, pp. 62–75.

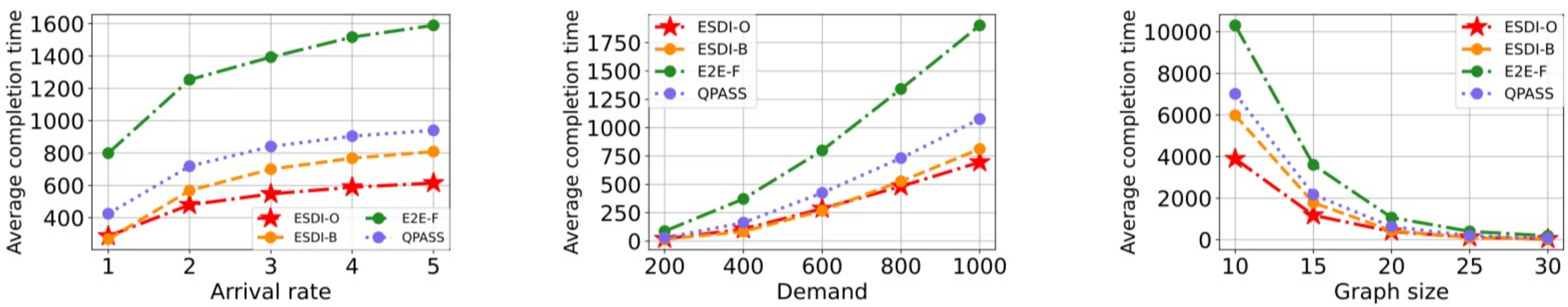
# Performance Evaluation




(a) Impact of the arrival rate (b) Impact of the demand (c) Impact of the graph size

Fig. 4: Success ratio between ESDI and state-of-the-art algorithms

**1) MRED with optimal EDR can significantly improve network-wide throughput.**
**2) Scheduling via prioritization (ESDI-E) can additionally finish more commodities before deadlines.**




(a) Impact of the arrival rate (b) Impact of the demand (c) Impact of the graph size

Fig. 5: Average completion time between ESDI and state-of-the-art algorithms

**Scheduling via prioritization (ESDI-O) can reduce average completion time for commodities without deadlines.**

# Performance Evaluation

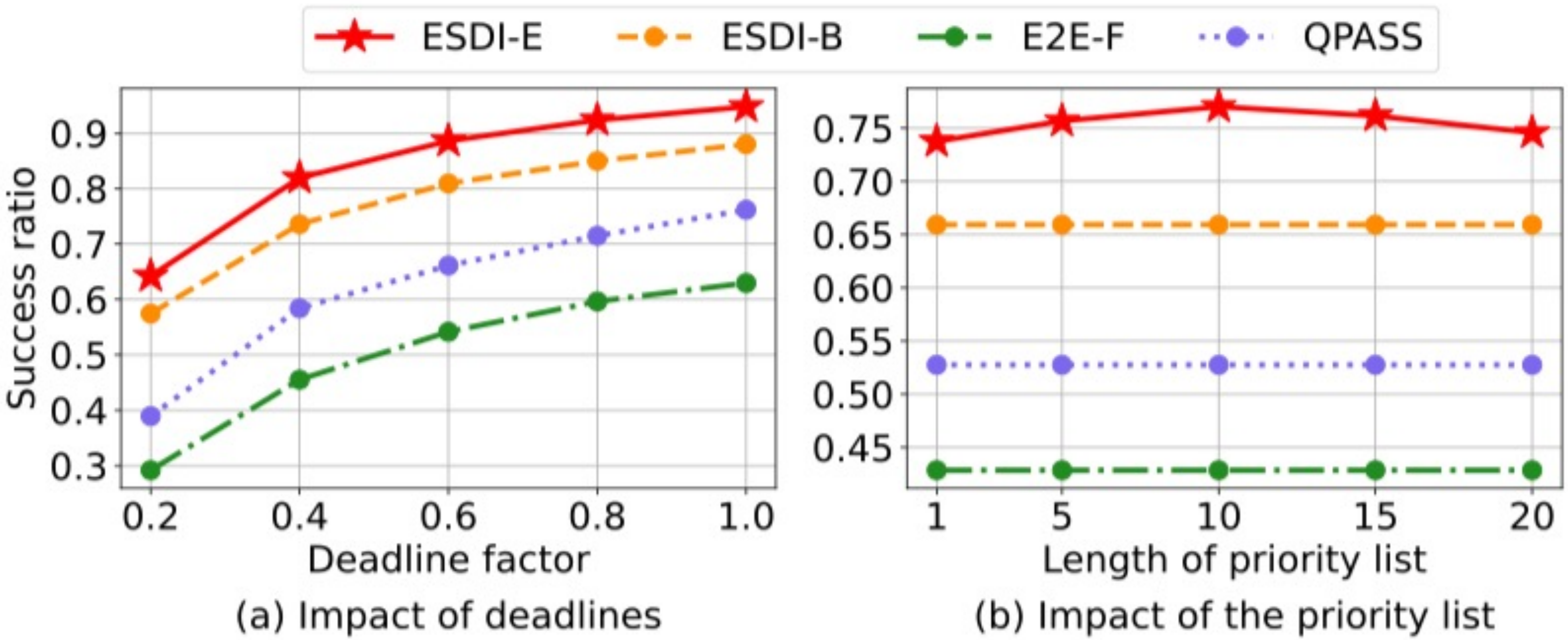



Fig. 6: Impact of deadlines and the length of priority list

**(6a): Increasing success ratios with increasing deadline factors**
**(6b): A trade-off between scheduling and work conservation**

# Outline

- Background and Motivation
- System Modeling
- Solution Design
- Performance Evaluation
- **Discussions, Future Work and Conclusions**

# Other Perspectives, Conclusions

- What we have done in this work
  - Entanglement scheduling and distribution
    +
    heterogeneous quantum applications } Modeling & solving

- What could be improved
  - Entanglement: throughput, fidelity, cost } Attributes Consideration
  - Entanglement source deployments
  - Layer quantum networks
  - Queueing-based buffer analysis } Network Design
  - Multi-hop fidelity improvements } Analysis and Algorithms

- **Conclusions:** design a comprehensive framework for the quantum Internet.

# Thank you very much!

Q&A?