Why Riding the Lightning? Equilibrium Analysis for Payment Hub Pricing

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Outlines

Outlines

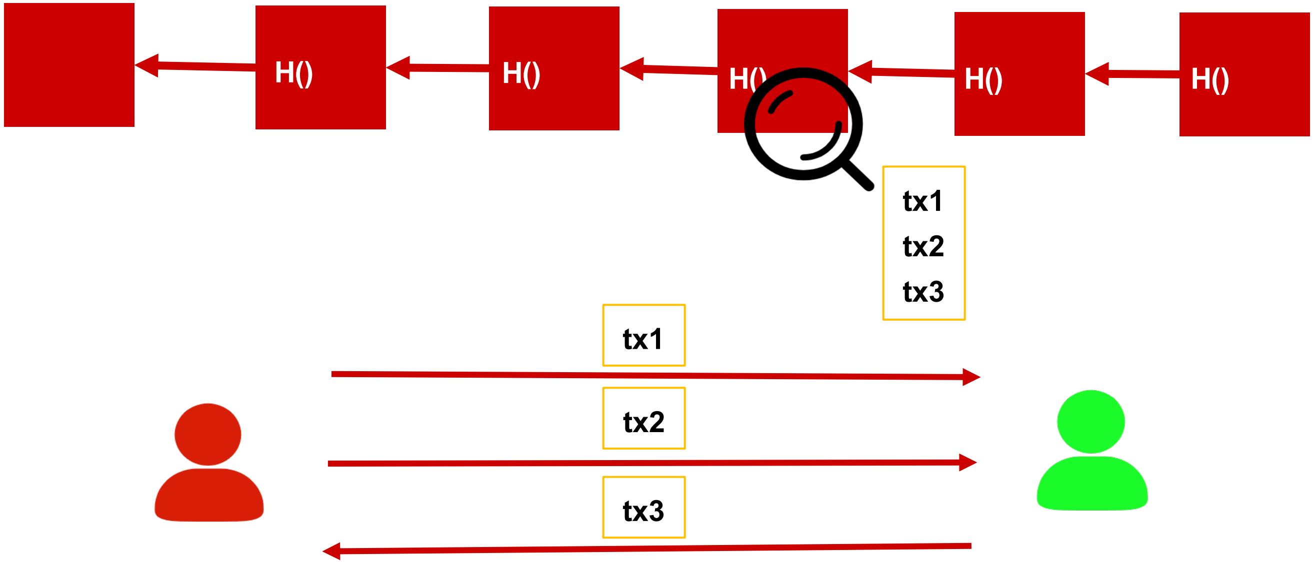
Scalability Problem of Bitcoin

7 tx/s

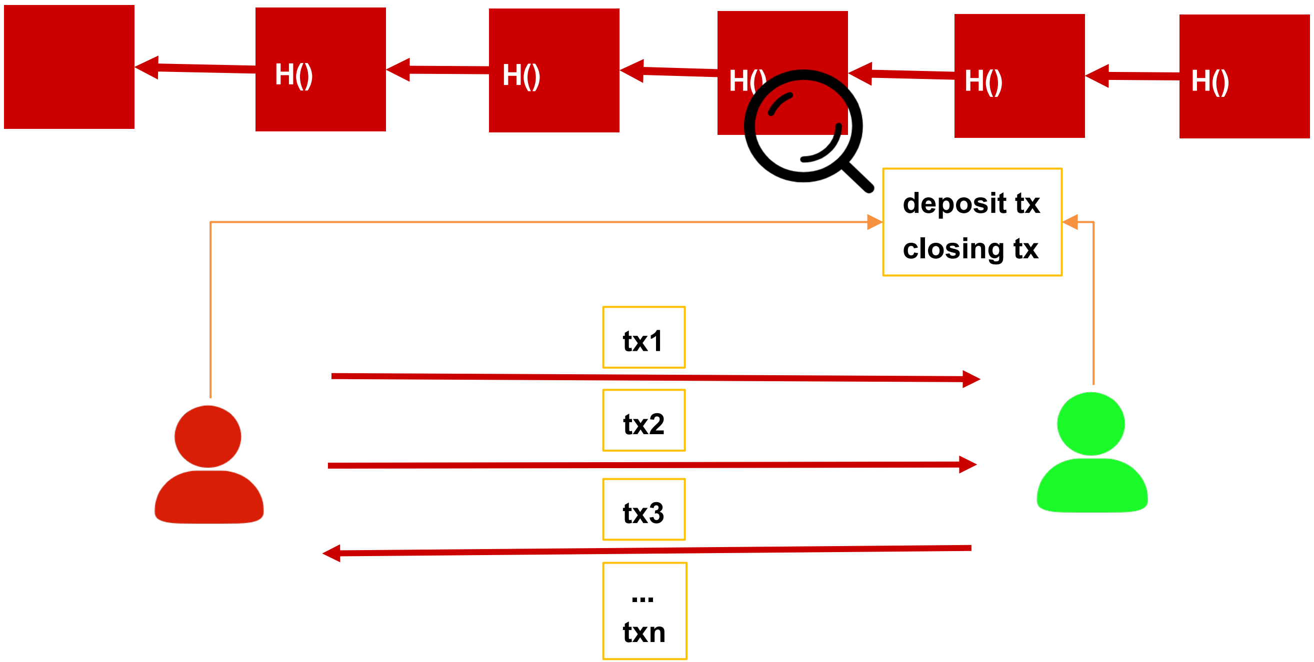
45,000 tx/s

Payment Channel

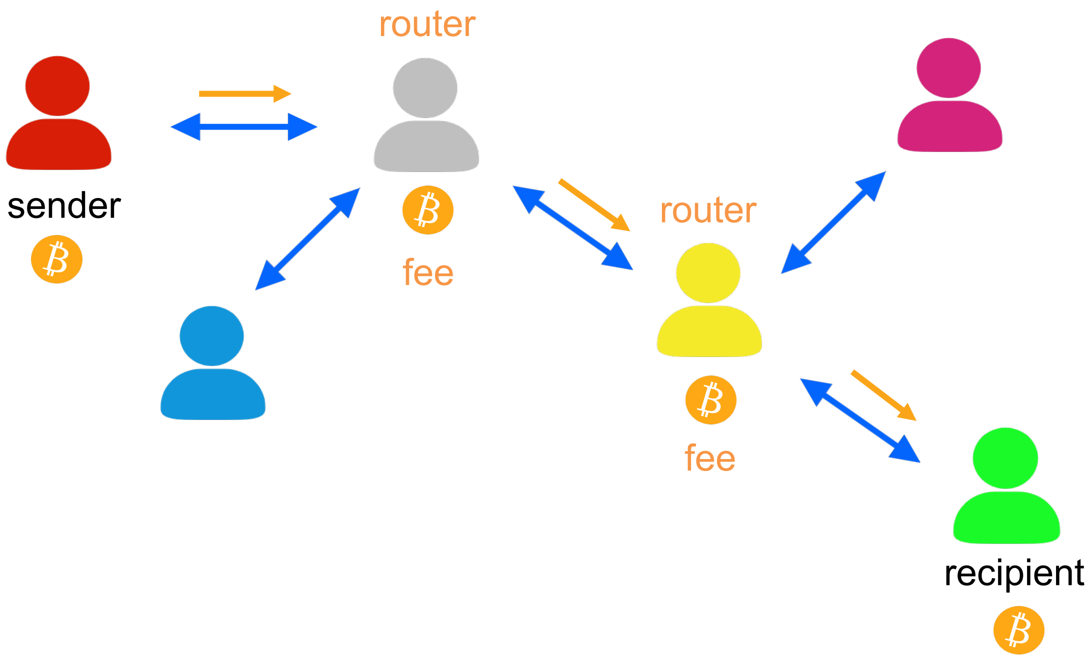
- Blockchain



Payment Channel



Payment Channel Network



Question Statement

- If routers can set fees freely,

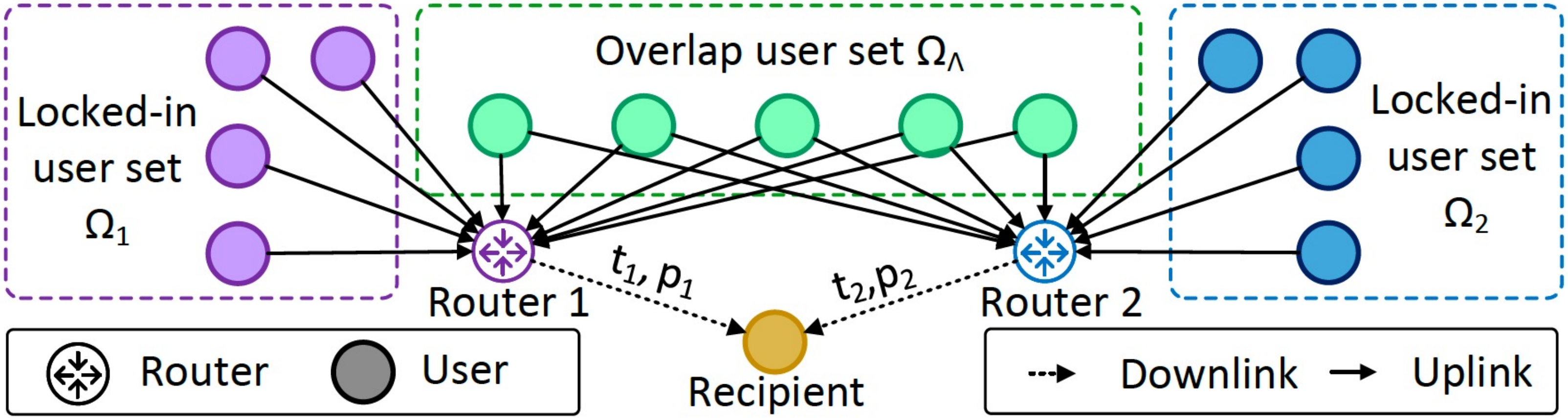
  will this lead to selfish fee setting that will increase PCN fees to be comparable to on-chain transaction fees, thus canceling the PCN's economic advantage?

Overview

- Equilibrium analysis
  - Two-hub model
  - Game between senders and routers
  - Existence of pure Nash Equilibriums (NEs)
  - Derive lower and upper bounds on the equilibrium revenue
- Algorithm to find all pure NE

Outlines

System model



Outlines

Price-Setting Game

- **Generalized Bertrand competition**
  - non-continuous demand curves
  - locked-in demand
  - capacity constraints

Strategy Space and Demand Function

- Strategy Space

$$Y = [RE, c_{\max}]$$

Reserved price

Maximum valuation

- Demand Function

$$d_\tau(p) = \sum_{k \in S_\tau(p)} \delta_k$$

Demand of sender *k*

Available user set

$$S_\tau(p) = \{k \in \Omega_\tau | c_k \geq p\}, \ \tau \in \{1, 2, \Lambda\}$$

- left-continuous and monotonically non-increasing step function

Utility Function

$$\Pi_i(p_i, p_{\neg i}) = \begin{cases} L_i(p_i) \equiv p_i \cdot \min\{t_i, d_i(p_i) + d_\Lambda(p_i)\}, & \text{if } p_i < p_{\neg i}, \\ \Phi_i(p) \equiv p \cdot \min\{t_i, d_i(p) + \phi_i d_\Lambda(p)\}, & \text{if } p_i = p_{\neg i} = p, \\ M_i(p_i) \equiv p_i \cdot \min\{t_i, d_i(p_i) + \psi_i\}, & \text{if } p_i > p_{\neg i}, \end{cases}$$

$$\phi_i = \frac{\max\{0, t_i - d_i(p)\}}{\max\{0, t_i - d_i(p)\} + t_{\neg i}} \qquad \psi_i = \max\{0, d_\Lambda(p_i) - t_{\neg i}\}$$

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Pure Nash Equilibrium (NE)

- Strategy Profile $(p_1^*, p_2^*) \in Y \times Y$

  $for\ \forall i \in \{1,2\},\ p_i^*\ is\ a$ best response $p_i^* \in BR_i(p_{\neg i}^*)$

- No router can unilaterally change its price to an alternative pure strategy and get a higher payoff

Bounds on Equilibrium Revenue

- Lower bound

Monopoly revenue

$$R^* \geq \sum_{i=1}^{2} \max_{p_i \in Y} \{M_i(p_i)\} = R^*_{\mathrm{LB}}$$

- Upper bound

$$R^* \leq \max_{p_1, p_2 \in Y} \{\sum_{i=1}^{2} \Pi_i(p_i, p_{\neg i})\} = R^*_{\mathrm{UB}}$$

NE Analysis

Theorem:

Best responses and pure NEs can only exist when both routers set prices <u>at the valuation or RE</u>.

Lemma:

Given the other router's price $p_{\neg i}$, the best response set is empty iff $sup_{p_i \in Y}\{\Pi_i(p_i, p_{\neg i})\} = L_{i(p_{\neg i})} > \Phi_i(p_{\neg i})$.

Pure NE Searching

- Find the candidate best response sets of two routers.
- Add strategy profile to the pure NE set
  - the best response sets of both routers exist given each other's price
  - prices of both routers are in their best response sets respectively

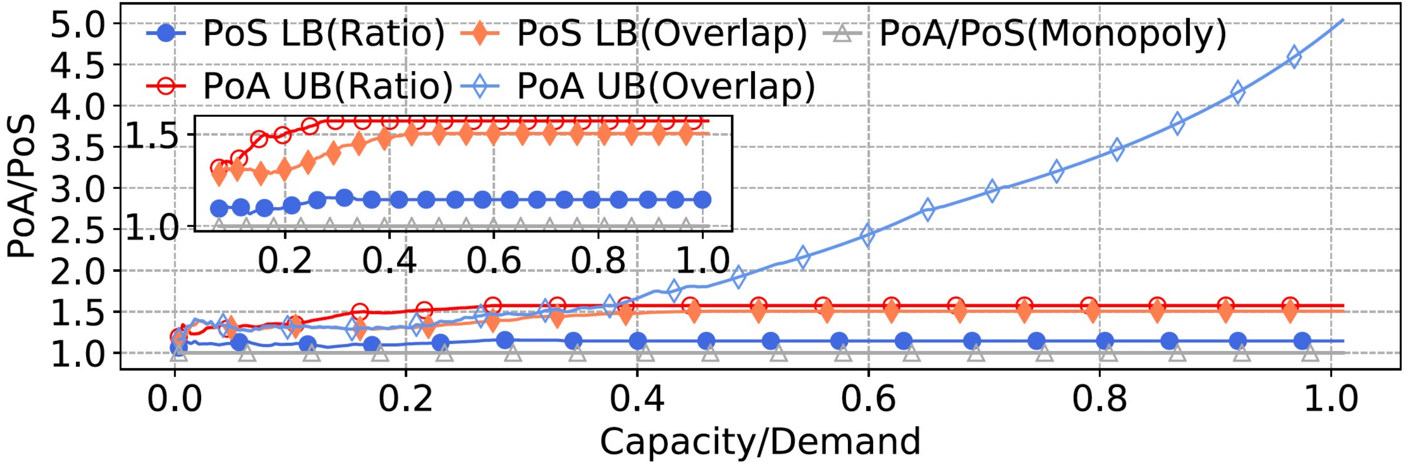
Outlines

Simulation Settings

- Lighting Network (LN) topology
- Routers and users
  - Choose the two most connected nodes as routers 1 and 2
    - 390 overlap users, 620 (496) locked-in users
  - Channel capacity: 10^6 satoshi
- Demand
  - Sampling transactions from a real-world credit card dataset[1]
- User distributions
  - Ratio, Overlap, Monopoly

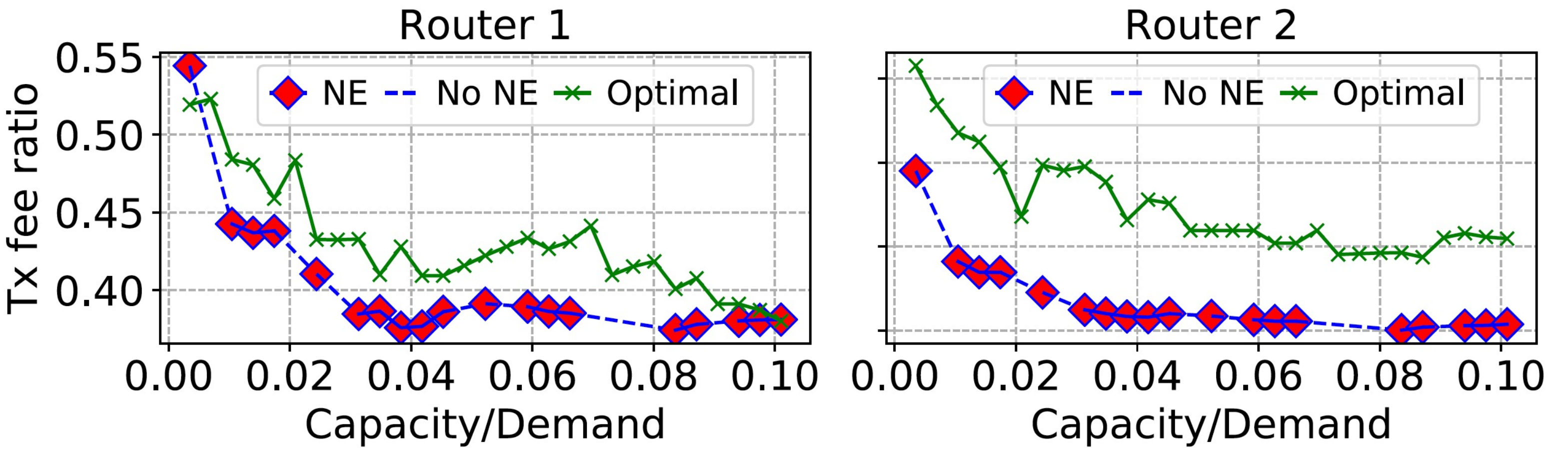
[1] “Credit Card Fraud Detection,” accessed 2021-11-12. [Online]. Available: https://www.kaggle.com/mlg-ulb/creditcardfraud

Simulation Results



Capacity ↑ Competition ↑

Simulation Results



PCN transaction fees can be driven down

Outlines

Our Conclusion

- The competitive nature of PCN will ultimately
  - make its transaction fee much lower than the blockchain
  - especially when the network capacity becomes larger and larger

Thank you very much!

Q&A?